The Sales of Small firms : a multidimensional analysis

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The Sales of Small Firms: a multidimensional analysis

Cessions de PME : une approche multidimensionnelle

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Abstract

This paper endogeneizes the security voting structure in an auction mechanism used to sell a small firm. The design of security voting structure allows the seller to choose between two objectives which are not mutually consistent. If the seller wants to maximize his revenue, he should retain some shares to benefit from the future dividends generated by the acquirer. At the opposite, if he wants to sell his firm to the most efficient candidate, he should sell all the shares.

Keywords: Security voting structure, auctions, small firms.

Résumé

Ce papier étudie une procédure particulière de vente de PME, l'open-bid, dans laquelle nous endogénéisons la structure en droits de vote des actifs financiers. Cette procédure de vente se rapproche d'une enchère ascendante. Le vendeur peut avoir deux objectifs divergents: maximiser son revenu et assurer la pérénité de son entreprise. Le premier objectif est atteint si le vendeur conserve une partie des parts de l'entreprise afin de tirer bénéfices des dividendes futurs rélisés par le repreneur. Le second objectif implique la vente de la totalité des parts de l'entreprise.

Mots-cles: structure en droits de vote, enchères, PME

JEL classification: D44; G34

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1 Introduction

The small firms sale, i.e. those held and managed by a single owner, is not a central theme in corporate finance. However, it is not a marginal practice. The sale can take two forms: a negotiation between the seller and the acquirer or a tender offer which gives the firm to the highest bidder. This paper deals with the latter case. The seller may have two objectives: the maximization of his expected revenue and the allocative efficiency. This latter objective is reached when the firm is sold to the most efficient candidate, i.e. the one who will maximize the firm's value in the future.

Over the two last decades, a specific auction mechanism, the open bid, emerged as a central technic of small firms sales. The open bid is equivalent to a "classic" auction: the seller, as a monopoler, faces some potential acquirers. He has the bargaining power. He decides when to sell, what to sell (shares number) and who may participate to the auction. In practice, two types of auction can be used: ascending auction or first price auction. The ascending auction seems to be the most commonly used.

This paper considers the firm's sale as a transfer of corporate control. Corporation's securities provide the holder with particular claims on the firm's income stream and particular voting rights. These securities can be designed in various ways: one share of a particular class may have a claim to vote which is larger or smaller than its claim to income. The transfer takes place when the acquirer receives more than half of votes. Selling only the votes is prohibited. A potential acquirer must pay for the dividends rights which give voting rights (yielding private benefits). Thus, we have to distinguish between two items for sale: the dividends rights and the control of the firm through the voting rights. Nevertheless, these two items are interdependent because the income stream depends on the identity of the acquirer who has taken control. We have to notice that a combinatorial auction would let the bidders bid simultaneously on voting rights, dividend rights and the bundle of both. But due to legal restrictions, only simple (non-combinatorial) auction can be used.

In this paper, we show how a seller can take into account the respective impact of voting rights and dividends in bidders valuation. Actually, even with a simple ascending auction, the choice of the optimal security voting structure reflects the interdependencies between voting rights/private benefits and dividends rights/public benefits. Hence, this optimal structure determines the willingness to pay of the bidder. It changes the relative weight of private and public benefits in the total valuation of the bundle of assets. We argue that from the seller point of view, the optimal structure may sometimes differ from the traditional one-share-one-vote structure.

The paper is organized as follows. Section 1 presents the model. In addition to the sale process, we introduce security voting structure. Following Grossman and Hart (1988), we use it as a tool which allows the seller to screen the candidates. Our model is closed to the At and Morand (2000) approach. They study the role of security voting structure when a government wants to privatize a company and faces asymmetric bidders. We depart from their analysis, considering that both private and public benefits are private information of the bidders. Section 2 derives the results. We identify the security voting structure which maximizes the expected revenue of the seller. We show that the one share-one vote structure is not always optimal from the seller's standpoint. However, it ensures that the most efficient candidate takes control, i.e. the candidate who will maximize the firm's value in the future. The literature on hostile takeovers has also discussed the role of security voting structure, notably Grossman and Hart (1988) and Harris and Raviv (1988). Our results mitigate the superiority of the one share-one vote structure shown by these authors. Section 3 concludes.

2 The model

The model considers a firm sold through an auction. It is owned by a single risk neutral shareholder. Following Grossman and Hart (1988), we shall concentrate on the case where the seller may only create two classes of shares. He specifies the fraction of dividends s^c and the fraction of total votes v^c, for $c = 1, 2$ to which class c is entitled. Without loss of generality, we consider that the superior voting stock is class 1, i.e. $v^1 > v^2$, and we assume $v^1 + v^2 = 1$ and $s^1 + s^2 = 1$.

A candidate must obtain at least the fraction $\alpha \in (0.5, 1]$ of votes to take control of the firm. Moreover, we assume that $(v^1, v^2, s^1, s^2, \alpha)$ are common knowledge.

We consider two candidates which we shall to refer as r and t. They are risk neutral. This model makes the traditional distinction between security benefits (discounted sum of dividends) denoted by y_r and y_t under the two managements and the private benefits of control, respectively b_r and b_t. The private benefits of control refer to benefits the acquirer obtain for himself. The security benefits refer to the total market value of the corporation's securities. We assume that r is more efficient than t iff $y_r > y_t$. We have to mention that the highest bidder may not be the most efficient candidate because of private benefits.

To focus on the impact of securities structure, we follow the assumptions of At and Morand (2000). We shall assume that the seller tenders the fraction x of the class 1 shares and the remainder $\left(\frac{\alpha - xv^1}{v^2}\right)$ from the class 2 shares. The fraction x is such that $xv^1 + v^2 \geqslant \alpha$. This package brings α votes.

The willingness to pay of a buyer m for the package is:

$$w_m = \Omega y_m + b_m \qquad for \quad m = r, t \tag{1}$$

where $\Omega = \left(xs^1 + s^2 \frac{\alpha - xv^1}{v^2}\right)$

The Ω-structure represents the amount of income rights attached to the voting rights majority. These income claims determine the cost of majority. Notice that when $v^2 = 0$ a buyer m buys only the class 1 shares and his willingness to pay becomes $w_m = \alpha s^1 y_m + b_m$. Moreover, under the one share-one vote structure $(s^1 = v^1 = 1)$ $\Omega = \alpha$. It implies that under the one share-one vote security voting structure, the seller can not obtain a Ω-structure such that $\Omega < 0.5$. Due to legal restrictions, unbundling votes, i.e. $(s^1 = 0; s^2 = 1; v^1 = 1; v^2 = 0)$, are ruled out. So, the most extreme structure is $(s^1 \to 0; s^2 \to 1; v^1 = 1; v^2 = 0)$, Ω tends to 0. The two bidders are interested only in the votes and so, bid for the class 1 shares. Their willingness to pay becomes $w_m = s^1 \alpha y_m + b_m \longrightarrow b_m$.

The seller obtains direct revenue from the auction $E(S)$ and indirect revenue from the fraction $(1 - \Omega)$ of shares kept. This fraction yields dividends generated by the winner of the auction. Let $E(y_v)$ represents the expected value of dividends under the winner control. The total revenue of the seller is so:

$$E(S) + (1 - \Omega)E(y_v) \tag{2}$$

We suppose that both public and private benefits are private information. The two potential acquirers are assumed symmetrical ex-ante. The private benefits of bidder $m = t, r$ are i.i.d over $[\underline{y}, \overline{y}]$ with a strictly positive density function $f_y(.)$. Similarly, public benefits are drawn independently from the same cumulative distribution function $F_B(.)$ on the interval $B = [\underline{b}, \overline{b}]$, with density $f_b(.)$. The probability density $f_b(b_m)$ is non-negative. So, the probability density of the package value is given by the following convolution function with $f_{\Omega y}(.)$ the density function of Ωy:

$$
\begin{aligned}
f_w(w) &= \int_{-\infty}^{+\infty} f_b(b) f_{\Omega y}(w-b)db \\
&= \frac{1}{\Omega}\int_{\underline{b}}^{\overline{b}} f_b(b) f_y(\frac{w-b}{\Omega})db
\end{aligned} \tag{3}
$$

Without loss of generality, we consider that the firm, i.e. the package of securities, is sold through an ascending auction. In our settings, with risk neutral, symmetric bidders and independent private value, every traditional auction mechanism is revenue-equivalent.

We have to add a further assumption. Actually, the shape of the convolution function is closely related to the comparison of $\left\|\underline{b},\overline{b}\right\|$ and $\left\|\underline{y},\bar{y}\right\|$. Recall that $w=\Omega y+b$. If $y\in[\underline{y},\overline{y}]$, $\Omega y\in[\Omega\underline{y},\Omega\overline{y}]$, with $f_{\Omega y}(x)=\frac{1}{\Omega}f_y(\frac{x}{\Omega})$. If we suppose that $\overline{y}-\underline{y}<\overline{b}-\underline{b}$, then $\Omega(\overline{y}-\underline{y})<\overline{b}-\underline{b}\ \forall\Omega\in(0,1]$. This hypothesis reflects that, even in an independent private value setting (i.e. where y_r is statistically independent of y_t), public benefits cannot radically differ between the two potential acquirers. The difference between bidders efficiency has just a marginal effect on the benefits. Nevertheless, private benefits may highly differ. Technically, this hypothesis involves a constant convolution density function, whatever the choice of Ω. So, we can derive the general density function of the willingness to pay for the bundle of assets:

$$
\begin{aligned}
\forall w &\in [\Omega\underline{y}+\underline{b},\Omega\overline{y}+\underline{b}], \\
f_w(w) &= f_w^1(w)=\frac{1}{\Omega}\int_{\underline{b}}^{w-\Omega\underline{y}} f_b(b)f_y(\frac{w-b}{\Omega})db
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\forall w &\in [\Omega\overline{y}+\underline{b},\Omega\underline{y}+\overline{b}], \\
f_w(w) &= f_w^2(w)=\frac{1}{\Omega}\int_{w-\Omega\overline{y}}^{w-\Omega\underline{y}} f_b(b)f_y(\frac{w-b}{\Omega})db
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\forall w &\in [\Omega\underline{y}+\overline{b},\Omega\overline{y}+\overline{b}], \\
f_w(w) &= f_w^3(w)=\frac{1}{\Omega}\int_{w-\Omega\overline{y}}^{\overline{b}} f_b(b)f_y(\frac{w-b}{\Omega})db
\end{aligned} \tag{6}
$$

3 Firm's sale revenue

In ascending auction, the bidder optimal strategy is to remain active as long as the price is not greater than the willingness to pay and the opponent remains active. So, the expected revenue of the auction corresponds to the expected willingness to pay of the second bidder. The density function of the rank two order statistic is $2f_w(.)\,[1-F_w(.)]$. We obtain:

$$E(S)=\int_{\Omega\underline{y}}^{\Omega\overline{y}+\overline{b}} 2wf_w(w)\,[1-F_w(w)]\,dw \tag{7}$$

With uniform distribution functions and $\underline{b}=0$, we have:

$$E(S)=\frac{20\overline{b}^3+30\overline{b}^2(\overline{y}+\underline{y})\Omega-5\overline{b}(\overline{y}+\underline{y})\Omega^2+(\overline{y}+\underline{y})^3\Omega^3}{60\overline{b}^2}$$

Notice that:

$$\frac{\partial E(S)}{\Omega}=\frac{30\overline{b}^2(\overline{y}+\underline{y})-10\overline{b}(\overline{y}+\underline{y})\Omega+3(\overline{y}+\underline{y})^3\Omega^2}{60\overline{b}^2}$$

The latter equation is positive $\forall\Omega\in(0,1]$. It is interesting to notice that with uniform distribution function and nul minimum private benefits, the sale revenue is maximized when $\Omega=1$. The seller sells all the shares.

4 The allocative efficiency

In this part, we shall derive the security voting structure which ensures the allocative efficiency. Let $2F_w(w)f_w(w)$ be the density function of the better valuation between the two bidders. We can so derive the expected cash flows of the winner. Since $w_v=\Omega y_v+b_v$, we have $y_v=\frac{w_v-b_v}{\Omega}$ and so:

$$\begin{aligned} E(y_v) &= \frac{2}{\Omega^3}\int_{\Omega\underline{y}+\underline{b}}^{\Omega\overline{y}+\overline{b}}\left(\int_{\underline{b}}^{\overline{b}}(w_v-b)f_b(b)f_y(\frac{w_v-b}{\Omega})db\right) \\ &\left(\int_{\Omega\underline{y}+\underline{b}}^{w_v}\int_{\underline{b}}^{\overline{b}}f_b(b)f_y(\frac{w-b}{\Omega})db\right)dw_v \end{aligned} \tag{8}$$

We cannot exhibit general results. Nevertheless with uniform distribution function and $\underline{b} = 0$, we have:

$$E(y_v) = \frac{30\overline{b}^2(\overline{y} - \underline{y}) + 10\overline{b}(\overline{y} - \underline{y})^2\Omega - 3(\overline{y} - \underline{y})^3\Omega^2}{60\overline{b}^2}$$

$$\frac{\partial E(y_v)}{\partial \Omega} = \frac{(\overline{y} - \underline{y})^2(5\overline{b} - 3\Omega(\overline{y} - \underline{y}))}{30\overline{b}^2} > 0 \ \forall \Omega \in (0, 1]$$

Proposition 1 *With uniform distribution function and* $\underline{b} = 0$*, allocative efficiency is maximized under* $\Omega = 1$*.*

This result is straightforward. The relative weight of cash flows in total valuation of the shares bundle is increasing with respect to Ω. This tends to minimize the probability of having a less efficient pretender with high private benefits, wich defeats a more efficient one with low private benefits.

5 The maximization of expected total revenue

We have shown that the firm's sale efficiency (the probability of attribution to the bidder with high public benefits) is obtained when the seller sales all the shares. This structure minimizes the relative weight of private benefits in the securities valuation and so maximizes efficiency of the sale. In ascending auction, the bidder with the highest willingness to pay will obtain the firm. Nevertheless, the seller cannot obtain more than the second valuation of the bundle of share. Hence, if the winning bidder is also the most efficient acquirer, then the seller is better off retaining part of the shares. He will so obtain future high dividends. But, as the fraction of shares retained by the seller increases, the probability of selling the firm to the more inefficient pretender increases because the relative weight of private benefits in the total valuation is increased. The choice of the security design maximizing the global revenue reflects this trade off.

Hence, in order to benefit from the dividends generated by the winner, the seller has to retain the fraction $(1 - \Omega)$ of shares. This fraction implies a negative impact of Ω in the total revenue of the seller. This total revenue may be written:

$$
\begin{aligned}
E(R) &= E(S) + (1-\Omega)E(y_v) \\
&= \int_{\Omega \underline{y}+\underline{b}}^{\Omega \overline{y}+\overline{b}} \left(w - \frac{\int_{\Omega \underline{y}+\underline{b}}^{w} F_w(s)ds}{F_w(w)}\right).2F_w(w)f_w(w)dw \\
&\quad +(1-\Omega)\frac{2}{\Omega^3} \int_{\Omega \underline{y}+\underline{b}}^{\Omega \overline{y}+\overline{b}} \left(\int_{\underline{b}}^{\overline{b}} (w_v - b) f_b(b) f_y\left(\frac{w_v - b}{\Omega}\right) db \right) \\
&\quad \cdot \left(\int_{\Omega \underline{y}+\underline{b}}^{w_v} \int_{\underline{b}}^{\overline{b}} f_b(b) f_y\left(\frac{w-b}{\Omega}\right) db \right) dw_v
\end{aligned} \tag{9}
$$

With uniform distribution function and $\underline{b} = 0$, we have:

$$
\begin{aligned}
E(R) &= \frac{20\overline{b}^3 + 30\overline{b}^2(\overline{y}+\underline{y})\Omega - 5\overline{b}(\overline{y}+\underline{y})\Omega^2 + (\overline{y}+\underline{y})^3\Omega^3}{60\overline{b}^2} \\
&\quad +(1-\Omega)\frac{30\overline{b}^2(\overline{y}-\underline{y}) + 10\overline{b}(\overline{y}-\underline{y})^2\Omega - 3(\overline{y}-\underline{y})^3\Omega^2}{60\overline{b}^2} \\
&= \frac{20\overline{b}^3 + 30\overline{b}^2(\overline{y}-\underline{y}) + 5\overline{b}(\overline{y}-\underline{y})^2(2-3\Omega)\Omega + (\overline{y}-\underline{y})^3\Omega^2(4\Omega-3)}{60\overline{b}^2}
\end{aligned}
$$

Maximizing the total revenue gives Ω^*, the optimal security structure. We have :

$$
\frac{\partial E(R)}{\partial \Omega} = \frac{2(\overline{y}-\underline{y})^2 \left[\overline{b}(5-15\Omega) + 3(\overline{y}-\underline{y})\Omega(2\Omega-1)\right]}{60\overline{b}^2}
$$

and:

$$
\frac{\partial^2 E(R)}{\partial \Omega^2} = \frac{(\overline{y}-\underline{y})^2 \left[-5\overline{b} + (\overline{y}-\underline{y})(4\Omega-1)\right]}{10\overline{b}^2} \leq 0 \ \forall \Omega \in (0,1]
$$

so, if $\exists \Omega \in (0,1]$ such that $\frac{\partial E(R)}{\partial \Omega} = 0$ then this is the optimal Ω-structure. We have:

$$
\Omega^* = \frac{15\overline{b} + 3(\overline{y}-\underline{y}) - \sqrt{9(5\overline{b}+\overline{y}-\underline{y})^2 - 120\overline{b}(\overline{y}-\underline{y})}}{12(\overline{y}-\underline{y})}
$$

Recall that if $\Omega^* < 0.5$, then the seller cannot use only one-share-one-vote structure. He has to issue two classes of shares characterized by different security voting structure. We can exhibit cases where the optimal structure departs from traditional one. Consider the following example:

Suppose $\overline{y} = 1050$, $\underline{y} = 1000$, $\overline{b} = 100$, *then:*

$$\Omega^* = \frac{33 - \sqrt{849}}{12} \simeq 0,32186$$

The optimal structure Ω^* *which maximizes total revenue, is strictly less than 0.5. It cannot be reached with traditional one share-one vote structure.*

This last counterexample allows us to have the following proposition:

Proposition 2 *When the candidates are symmetrical and both private and public benefits are private informations, there exists some cases in which the seller should issue several classes of shares with one at least richer in votes. This structure differs from the efficiency-maximizing structure, which is reached under the one share-one vote structure and the vote rule* $\alpha = 1$ **.**

Remark 1 $\bar{y} = \underline{y}$, *i.e. no uncertainty on public benefits, yields* $\frac{\partial E(R)}{\partial \Omega} = 0$. *The global revenue of the seller is independent of the security design.*

6 Conclusion

A small firm's seller may have two distinct objectives which are not mutually consistent. He may want to maximize the sale's revenue. But he may want to ensure himself that his firm will be well managed. This paper endogeneizes the security voting structure in an auction mechanism used to sell the firm. Thus, by designing a specific security voting structure, the seller may choose between the two objectives. If the seller wants to maximize his revenue, he should keep some shares. The security voting structure design trades off between sell all the shares and retain some shares to benefit from the future dividends generated by the winner. At the opposite, if he wants to sell his firm to the most efficient candidate, he should sell all the assets.

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